1. You have a 5-by-5 grid of squares into which you can place circles. Each square of the grid can hold either one or zero circles.

   (a) If you take a single circle and place it at random onto the grid four times, **removing** the circle before placing it each time, what is the probability that you place the circle in a corner of the grid at least once?

   (b) You have four indistinguishable red circles and three indistinguishable blue circles. How many distinct ways can all seven circles be placed in the grid?

2. Evaluate $\langle x \rangle$ and $\langle \delta x^2 \rangle$ for the following distributions:

   (a) Random variable X is the number of heads for flipping a biased coin N times, where the probability of landing on heads is p.

   (b) This continuous probability distribution, where N is the normalization constant:

$$p(x) = N \exp\left[-\frac{x^2 + ax}{2\sigma^2}\right]. \tag{1}$$

   Hint: Recall that

$$\int_{-\infty}^{\infty} \mathrm{d}x \, e^{-a(x+b)^2} = \sqrt{\frac{\pi}{a}}.$$

3. You perform an experiment. Let X represent the random variable corresponding to your N data points.

   (a) Writing down an equation, what is the mean of the sample data $\langle X \rangle$? What is the variance $\langle \delta X^2 \rangle$?

   (b) For a finite number N of measurements in each experiment, the sample mean $\langle X \rangle$ is also a random variable. To avoid confusion, let's call the sample mean $\bar{X}_N$. It has its own probability distribution and therefore also its own mean $\langle \bar{X}_N \rangle$ and variance $\langle \delta \bar{X}_N^2 \rangle$. Find them symbolically.

   (c) Describe the relationship between the distribution of the random variable X and the random variable $\bar{X}_N$. How does the distribution of $\bar{X}_N$ change as $N \to \infty$?

4. Find $\langle x^3 \rangle$ and $\langle \delta(x - \mu) \rangle$, where μ is the distribution mean, for the following distributions. Remember to normalize your distributions.

   (a) The uniform distribution, in which the probability of any value x between a and b is equal ($a \leq x \leq b$).

   (b) The probability distribution given by

$$p(x) = N \exp\left[-\frac{x^2 + ax}{2\sigma^2}\right] \tag{2}$$

   for which you should have already found the mean and the variance, for the case that a is zero.

5. *Bonus Question:* Evaluate μ and σ^2 for the Poisson distribution, which is a discrete probability distribution given by $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, where λ is a parameter of the distribution, and $x = 0, 1, 2, \ldots, \infty$.

   Hint:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$