1. Given the orthonormal basis vectors from before,

$$|v_B\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |v_D\rangle = \begin{bmatrix}0\\1\end{bmatrix} \tag{1}$$

evaluate $\sum_{i=1}^{n} |v_i\rangle \langle v_i|$ where $\{|v_1\rangle \ldots |v_n\rangle\}$ are the basis vectors and $|\cdot\rangle \langle\cdot|$ is the outer product. What do you notice about the result?

2. Suppose we have an nxn matrix $\mathbf{C}$, an nx1 column vector $|x\rangle$ and a 1xn row vector $\langle y|$. Predict the type of result (scalar, matrix, bra, or ket) for each operation:

   (a) $\langle y|x\rangle$
   (b) $\mathbf{C}\,|x\rangle$
   (c) $\langle y|\,\mathbf{C}$
   (d) $|x\rangle \langle y|$
   (e) $\langle y|\,\mathbf{C}\,|x\rangle$

3. Given the matrices

$$\mathbf{A} = \begin{bmatrix}0 & i\\3 & 3\end{bmatrix} \quad \mathbf{B} = \begin{bmatrix}3 & -1\\2 & i\end{bmatrix}$$

perform the following operations:

   (a) $\mathbf{AB}$
   (b) $\mathbf{BA}$
   (c) Are your answers from a) and b) the same? Should they be?

4. Consider the matrix

$$\mathbf{D} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & -3 & 0\\0 & -2 & 0 & 0\\0 & 0 & 0 & 5\end{bmatrix}.$$

   (a) Does this matrix have any eigenvalue(s) that can be determined by inspection? If so, specify the eigenvalue(s) and corresponding eigenvector(s).
   (b) Determine the remaining eigenvalues and eigenvectors for this matrix. (Hint: consider how the results from (a) might be used to simplify the problem.)

5. Use summation notation to show that the trace of the matrix product $\mathbf{AB}$ is equal to the trace of the matrix product $\mathbf{BA}$ for any two square matrices $\mathbf{A}$ and $\mathbf{B}$ of (the same) arbitrary size.

6. (Bonus). Matrix $\mathbf{F}$ has eigenvalues $\lambda = -1, \sqrt{3}, 2$ corresponding to the orthonormal eigenvectors $|a\rangle, |b\rangle, |c\rangle$, respectively.

   (a) Given the power series $e^{\mathbf{X}} = \sum_{k=0}^{\infty} \frac{1}{k!}\mathbf{X}^k$ for an $n\times n$ real or complex matrix $\mathbf{X}$ (where $\mathbf{X}^0 = \mathbf{1}_n$), expand $e^{i\mathbf{F}t}$. Examine the action of each of the first few terms on an eigenvector $|v\rangle$ and use this result to re-collapse the series.

(b) Using your result from part (a) and given the normalized vector

$$|\chi\rangle = \frac{1}{\sqrt{7}}|a\rangle - \frac{2}{\sqrt{7}}|b\rangle + \frac{i+1}{\sqrt{7}}|c\rangle$$

find $|\Phi(t)\rangle = e^{i\mathbf{F}t}|\chi\rangle$.

(c) Set up the calculation for $|\Phi(t)\rangle = e^{i\mathbf{F}t}|\chi\rangle$ explicitly in matrix form.