General Techniques

1. Solve using an integrating factor:

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{2}{x^2}$$

Fourier Series

2. Find the solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 2y = \frac{1}{2}\sin(2x)$ over the region $0 \leq x \leq 2\pi$ by assuming your solution can take the form of a Fourier series. This problem was taken from this differential equations resource.

3. By assuming your solution can take the form of a Fourier series over the region $0 \leq x < 2$ find the solution to

$$\frac{d^2y}{dx^2} + \omega^2 y = \begin{cases} 1 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \end{cases}$$

   ω is a constant, you may leave your solution in terms of this value.

Power series

4. Solve $(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 0$ by assuming the solution takes the form of a power series centered at x=0. You will not be able to find numerical values for the coefficients, you just need to find equations that relate the coefficients of higher powers of x to coefficients for lower powers of x. (Find what c_{n+2} is in terms of n and c_n, for example.)

5. Solve $\frac{dy}{dx} - 2y + \cos(x) = 0$ by assuming the solution takes the form of a power series centered at x=0. You will not be able to find numerical values for the coefficients, you just need to find equations that relate the coefficients of higher powers of x to coefficients for lower powers of x. (Find what c_{n+1} is in terms of n and c_n, for example.)